Método de Cramer

Sistemas de Ecuaciones Lineales

Sirve para resolver problemas del tipo:

$$
\begin{aligned}
x + y + z &= 11 \\
2x - y + z &= 5 \\
3x + 2y + z &= 24
\end{aligned}
$$

$$
A = \begin{matrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{matrix} \qquad b = \begin{matrix} 11 \\ 5 \\ 24 \end{matrix}
$$

Para aplicarlo se debe cumplir que :

- ✓ *Número de incógnitas = Número de ecuaciones*
- ✓ *Det (A)≠ 0*

Procedimiento:

Calcular los siguientes determinantes:

$$A = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 11 \\ 2 & -1 & 1 & 5 \\ 3 & 2 & 1 & 24 \end{array}\right)$$

$$|Ax| = \begin{vmatrix} 11 & 1 & 1 \\ 5 & -1 & 1 \\ 24 & 2 & 1 \end{vmatrix} \qquad |Ay| = \begin{vmatrix} 1 & 11 & 1 \\ 2 & 5 & 1 \\ 3 & 24 & 1 \end{vmatrix} \qquad |Az| = \begin{vmatrix} 1 & 1 & 11 \\ 2 & -1 & 5 \\ 3 & 2 & 24 \end{vmatrix}$$

Solución:

$$x = \frac{|Ax|}{|A|}$$

$$y = \frac{|Ay|}{|A|}$$

$$z = \frac{|Az|}{|A|}$$

Ejemplo

Resolver el siguiente SEL:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 2 \\ -1 & 1 & 2 & 3 \\ 1 & -1 & 1 & -1 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

p1 de clase auxiliar vista el 28 de marzo

$$x1 = \frac{\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 2 \\ 1 & 1 & 2 & 3 \\ 1 & -1 & 1 & -1 \end{vmatrix}}{|A|}$$

$$x2 = \frac{\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 2 \\ -1 & 1 & 2 & 3 \\ 1 & 1 & 1 & -1 \end{vmatrix}}{|A|}$$

$$x3 = \frac{\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ -1 & 1 & 1 & 3 \\ 1 & -1 & 1 & -1 \end{vmatrix}}{|A|}$$

$$x4 = \frac{\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 1 \\ -1 & 1 & 2 & 1 \\ 1 & -1 & 1 & 1 \end{vmatrix}}{|A|}$$

Otro Método (Gauss-Jordan):

i. $$A^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 2 \\ -1 & 1 & 2 & 3 \\ 1 & -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & -2 & 0 & -2 \end{bmatrix};$$

ii. $$A^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & -2 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 7 & 2 \\ 0 & 0 & -4 & 0 \end{bmatrix};$$

iii. $$A^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{4}{7} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 7 & 2 \\ 0 & 0 & -4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & \frac{8}{7} \end{bmatrix}$$

iv. $m_{21} = -1$ $m_{31} = 1$ $m_{41} = -1$; $m_{32} = -2$ $m_{42} = 2$; $m_{43} = \frac{4}{7}$

v. $$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 1 & -2 & -\frac{4}{7} & 1 \end{bmatrix}; U = A^3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & \frac{8}{7} \end{bmatrix}$$

i. $Ly = b \iff \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 1 & -2 & -\frac{4}{7} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

ii. $y = \begin{bmatrix} 1 \\ 0 \\ 2 \\ \frac{8}{7} \end{bmatrix}$

iii. $Ux = y \iff \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & \frac{8}{7} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ \frac{8}{7} \end{bmatrix}$

iv. $x_n = \frac{u_{n(n+1)}}{u_{nn}} \iff x_4 = 1$

v. $x_k = \frac{1}{u_{kk}} \left(u_{k(n+1)} - \sum_{j=k+1}^{n} u_{kj} x_j \right) \; k = (n-1), ..., 1$

A. $k = 3 \iff x_3 = \frac{1}{u_{33}} \left(y_3 - [u_{34} x_4] \right) = \frac{1}{7} \left(2 - [2 \times 1] \right) = 0$

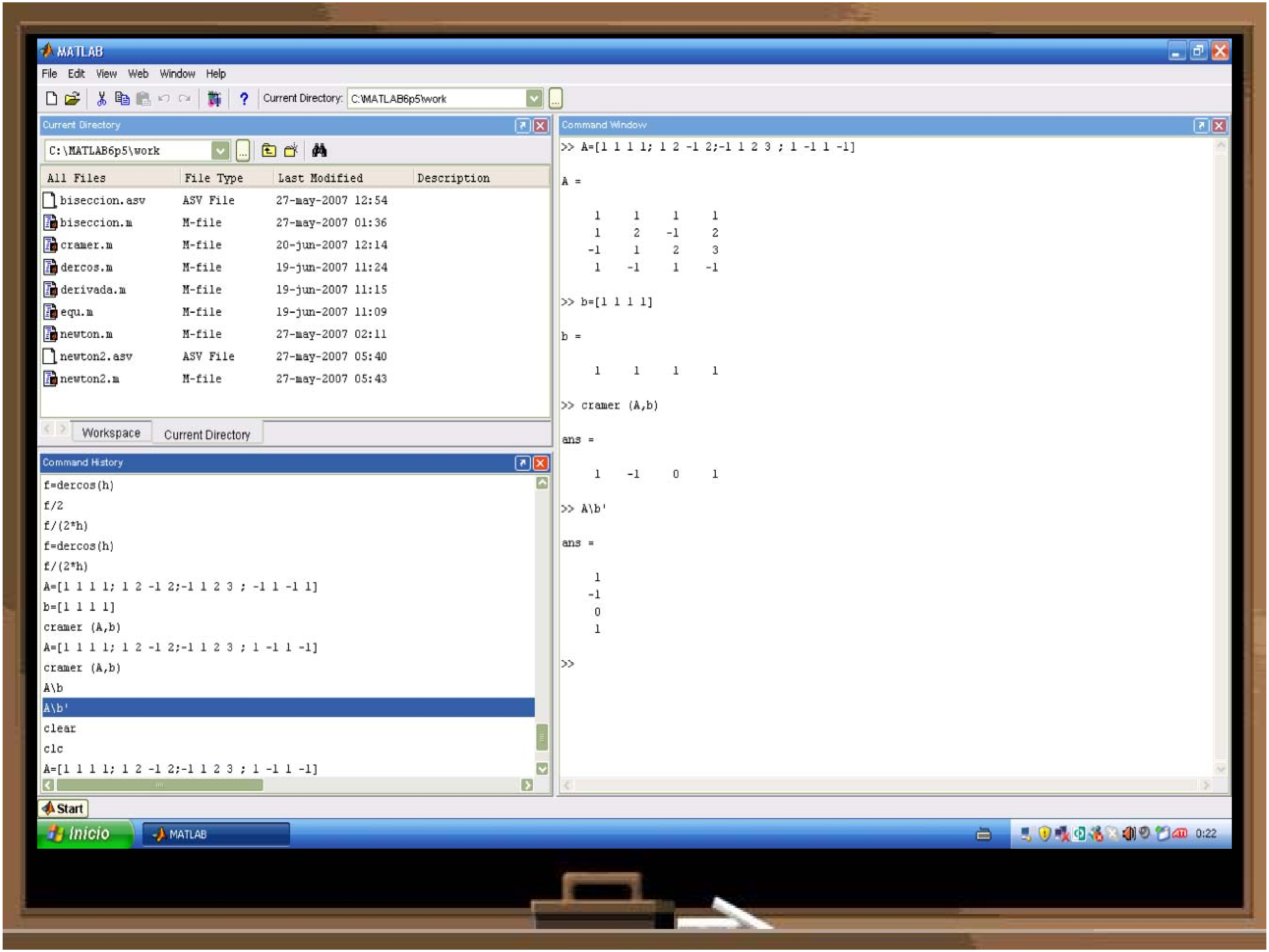
B. $k = 2 \iff x_2 = \frac{1}{u_{22}} \left(y_2 - [u_{23} x_3 + u_{24} x_4] \right) = \frac{1}{1} \left(0 - [(-2) \times 0 + 1 \times 1] \right) = -1$

C. $k = 1 \iff x_1 = \frac{1}{u_{11}} \left(y_1 - [u_{12} x_2 + u_{13} x_3 + u_{14} x_4] \right) = \frac{1}{1} \left(1 - [(-1) \times 1 + 1 \times 0 + 1 \times 1] \right) = 1$

vi. $x = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

En Matlab

```
function res = cramer (A,b)

n = length(b); %largo del vector
denominador = det(A);
C = A; %guardo valor original de matriz A
b = b'; %transpongo b para adecuar dimensiones

for i = 1:(n)

    C(:,i)= b; %reemplazo columna i-esima por vector b
    incognita(i) = det(C)/denominador; %solucion
    C = A; %vuelvo a la matriz original

end %fin del for

res = incognita;

end %fin de la función
```


MATLAB
File Edit View Web Window Help
Current Directory: C:\MATLAB6p5\work
Current Directory
C:\MATLAB6p5\work
All Files File Type Last Modified Description
biseccion.asv ASV File 27-may-2007 12:54
biseccion.m M-file 27-may-2007 01:36
cramer.m M-file 20-jun-2007 12:14
dercos.m M-file 19-jun-2007 11:24
derivada.m M-file 19-jun-2007 11:15
equ.m M-file 19-jun-2007 11:09
newton.m M-file 27-may-2007 02:11
newton2.asv ASV File 27-may-2007 05:40
newton2.m M-file 27-may-2007 05:43
Workspace
Current Directory
Command History
f=dercos(h)
f/2
f/(2*h)
f=dercos(h)
f/(2*h)
A=[1 1 1 1; 1 2 -1 2;-1 1 2 3 ; -1 1 -1 1]
b=[1 1 1 1]
cramer (A,b)
A=[1 1 1 1; 1 2 -1 2;-1 1 2 3 ; 1 -1 1 -1]
cramer (A,b)
A\b
A\b'
clear
clc
A=[1 1 1 1; 1 2 -1 2;-1 1 2 3 ; 1 -1 1 -1]
Command Window
>> A=[1 1 1 1; 1 2 -1 2;-1 1 2 3 ; 1 -1 1 -1]
A =
1 1 1 1
1 2 -1 2
-1 1 2 3
1 -1 1 -1
>> b=[1 1 1 1]
b =
1 1 1 1
>> cramer (A,b)
ans =
1 -1 0 1
>> A\b'
ans =
1
-1
0
1
>>
Start
Inicio
MATLAB
0:22

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FIN